Coherence and frequency spectrum of a Nd:YAG laser -
generation and observation devices.

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\section*{ABSTRACT}
The coherence of a Nd:YAG CW laser is analyzed using a Michelson interferometer. Fringe contrast is measured as the path difference is varied by changing the length of one arm. The fringe contrast, as expected, is maximum when there is no path difference between arms. However, the fringe contrast does not decrease monotonically. It decreases and then increases several times before fading away. This behaviour is reminiscent of the fringe contrast depending on aperture and the uncovering of the Fresnel zones. In order to evaluate the mode structure it is necessary to consider the geometric parameters and Q factor of the cavity, the medium gain curve and the type of broadening.

The non interference of waves principle requires that two (or more) modes competition or their interference can only take place through matter non linear interaction. Therefore, and in addition, it is important to consider the setup and type of detectors employed to monitor the frequency and/or time dependence. In as much as speckle is recognized as an interference phenomenon taking place at the detector plane, say the retina, the role of the sensing element in the detection of mode beats should also be decisive.

\section*{1. INTRODUCTION}
The concept of coherence is anchored to the correlation of the electromagnetic field at two points in time-space: If the field amplitude and phase are known at one position or region at a certain time, with what certainty can the field amplitude and phase be obtained at another position and time. This problem can become very complicated if several fields are present in the space-time region under consideration.

However, the theory of coherence involves several other assumptions that are more or less explicit: The emphasis is always on the light source. Incoherent sources are often addressed by invoking the Van Cittert–Zernike theorem.\textsuperscript{1} Two-beam fringe visibility due to a light beam of broad spectral distribution is given by the autocorrelation Wiener-Khintchine theorem. Coherence of laser sources are solely described in terms of the laser operating parameters. Rarely are the observers’ conditions or the nature of its measurements treated on an equal footing. It is well known that the coherence from a given source can be reduced (or modified) if the light is filtered. This can be achieved in a variety of ways such as spatial filtering that involve passing the light through small pinholes. Temporal coherence can be increased using tinted glass or interference filters. Nonetheless, the artifacts on the observers’ side are often played down.

On the other hand, tremendous confidence has been given to the equivalence of time and frequency measurements. Interference of light, say in Fabry-Perot interferometers involve the correlation of the field in very short times. Namely, the time required by light to travel back and forth between the Fabry-Perot mirrors. In a Michelson interferometer, the path difference between arms is usually quite small, and hence the time correlation is small although the measurement is actually performed in a much longer time. On the other hand, prism or diffraction grating spectrometers spatially separate different frequencies that are thereafter detected. Detection times are usually rather long and involve an averaging that should be acknowledged.

In the present manuscript, we study the modes of a Nd:YAG laser source. We observe the mode structure with a Michelson interferometer as well as with a grating monochromator. A Nd:YAG laser pointer was used as

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a laser source in order to have a very small cavity. In such cavities the modes are sufficiently separated to be
detected with a monochromator. A typical laser pointer setup downloaded from the internet is shown in figure 1.

Figure 1: Nd:YAG laser pointer structure. The resonator cavity is between the HR and OC mirrors. The cavity
has the Nd:YAG crystal and the intracavity frequency doubling KTP crystal.

2. SPECTROMETRY

The cavity has been chosen very small so that the longitudinal mode separation becomes large and discernible.
Recall that the modes contain an integer wavelength number

\[ m\lambda_m = 2L, \quad (m+1)\lambda_{m+1} = 2L \]

where \( m \) is an integer number, \( L \) is the cavity length and \( \lambda \) is the wavelength. Eliminate \( m \) from the two
contiguous modes relationships

\[ \frac{2L}{\lambda_m} = \frac{2L}{\lambda_{m+1}} - 1 \]

thus

\[ \frac{\lambda_m - \lambda_{m+1}}{\lambda_{m+1}\lambda_m} = \frac{1}{2L} \]

that is usually rewritten as

\[ \Delta\lambda = \frac{\lambda^2}{2L} \]

where \( \Delta\lambda = \lambda_m - \lambda_{m+1} \) and \( \lambda_{m+1}\lambda_m \approx \lambda^2 \). For a 1064 nm wavelength and 1 cm cavity

\[ \Delta\lambda = \frac{\lambda^2}{2L} = \frac{(1.064 \times 10^{-6})^2}{2 \times 10^{-2}} = 5.618 \times 10^{-11} \text{ m}. \]

This mode separation can just be resolved with a conventional 1 m focal length Czerny-Turner spectrometer
with 0.1 A resolution.

2.1 Experimental results

The Nd:YAG non-focussed output was steered via a multimode optical fiber 400 \( \mu \text{m} \) diameter into the spectrometer entrance slit. The light at the output slit was detected with a photodiode whose signal was subsequently amplified 70 dB. The 1 m focal length spectrometer\(^*\) has a 1200 1/mm grating. The system resolution is
\( 1 \times 10^{-11} \text{ m} \) when the slit widths are sufficiently closed. The spectrum revealed five modes as seen in figure 2.
Figure 2: Nd:YAG output wavelength scan

<table>
<thead>
<tr>
<th>Mode</th>
<th>λ (10⁻¹⁰m)</th>
<th>Δλ between maxima</th>
<th>Peak width</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5320.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5321.33</td>
<td>1.25</td>
<td>0.19</td>
</tr>
<tr>
<td>3</td>
<td>5322.54</td>
<td>1.205</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5325.63</td>
<td>3.09</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5326.88</td>
<td>1.25</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Mode separation from wavelength spectrum in figure 2.

The main features are shown in table 1.

Modes are separated by 1.25 ± 0.1 x 10⁻¹⁰m. However, recall that this spectrum is for the intracavity frequency doubled modes. The laser fundamental at twice the wavelength, modes are separated by 2.50 x 10⁻¹⁰. The cavity length obtained from this measurement is

\[ L = \frac{\lambda^2}{2\Delta\lambda} = \frac{(1.064 \times 10^{-6})^2}{2 \times 2.50 \times 10^{-10}} = 2.26 \times 10^{-3} \text{m}. \]

The cavity length is then somewhat shorter than our original 1 cm estimate. The first three modes are equally spaced and show a monotonic intensity increase. So far, it is almost a textbook image. However, there is a large separation before two other modes appear. This separation is larger than two modes but smaller than three modes 2.5<3.09<3.6, so they do not fit in the sequence. Nonetheless, the separation between these two modes is again the same 1.25 angstroms.

### 3. MICHELSON INTERFEROMETER

Let us now turn to the time analysis. The frequency \( \nu = \frac{c}{\lambda} \), from (1) is

\[ \Delta\nu = \nu_{m+1} - \nu_m = \frac{c}{2L}. \]

If we assume a 2 cm cavity, the frequency interval between modes is

\[ \Delta\nu = \frac{c}{2L} = \frac{3 \times 10^8}{2 \times 10^{-2}} = 1.5 \times 10^{10}. \]

The mode separation is then 15 GHz. The beat period should then be

\[ \tau = \frac{1}{\Delta\nu} = 66 \text{ ps}. \]

A Michelson interferometer was used to evaluate the coherence of the laser output. The laser output was magnified with a telescope arrangement. This beam is incident onto a beam splitter whose reflective coating is

Spex 1240
on the rear side. The reflected beam thus passes through the glass substrate after reflection from the BS coating. This beam is then incident on mirror M1 whose distance to the beam splitter can be adjusted. After reflection on M1 this beam passes again through the BS substrate before transmission and exiting the interferometer. Thus the beam on path 1 goes through the BS substrate two times after reflection. On the other hand, the beam transmitted through the BS goes through a piece of glass of the same material, thickness and inclination of the BS but without coating. It is then reflected on mirror M2 and passes back through the compensating glass to be reflected on the BS (without passing through its substrate. Thus, beam on path 2 passes through the glass plate twice. The two beams thus go through the same amount of material although via two similar but distinct optical elements.

![Diagram](image)

Figure 3: Michelson interferometer. T - expanding telescope, BS - beam splitter, CP - glass compensating plate, M2 - mirror, M1 mirror with translation stage, D - detection screen.

The beam splitter is aluminum coated. It has the advantage of having a very flat response for different frequencies. However, absorption in the BS becomes important. The intensity at the interferometer output was measured for each arm while blocking the other arm. The average of 20 measurements in arbitrary units is

\[ I_1 = 83.25 \]
\[ I_2 = 157.95 \]

The two arms intensity ratio at the output is \( I_2/I_1 = 1.897 \).

The mirror M1 with translation stage can be moved with a micrometer that is mounted on an arm that reduces the travelling length of the mirror. The mirror can be moved beyond the micrometer distance by placing two slabs either in front or behind the mirror.

### 3.1 Mirror M1 displacement calibration

In order to calibrate the mirror displacement with respect to the micrometer reading interference maxima were measured. The micrometer readings are in 10 \( \mu \) steps and a whole turn moves 500 \( \mu = 0.5 \text{mm} \). The light beam travels twice the mirror displacement \( 'd' \) since it travels that distance back and forth. The number of maxima \( m_{max} \) times the wavelength equals the distance travelled

\[ 2d = m_{max} \lambda \]

The micrometer was moved by hand ten steps (100\( \mu \)) and the number of maxima were recorded in a digital conversion program. The number of maxima were counted and the result is for several runs is shown in table 2. The average value is 360.85. Since the Nd:YAG doubled wavelength is 532 nm, the mirror displacement is
So the arm linking the micrometer to the mirror produces a reduction of 5.21, so that one micrometer step of 10\( \mu \) produces a mirror displacement of 1.92\( \mu \).

\[
d(500\mu) = \frac{360.85 \times 0.532}{2} = 95.98\mu
\]  

So the arm linking the micrometer to the mirror produces a reduction of 5.21, so that one micrometer step of 10\( \mu \) produces a mirror displacement of 1.92\( \mu \).

<table>
<thead>
<tr>
<th>experiment</th>
<th>Number of maxima</th>
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<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
<td>372</td>
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<tr>
<td>3</td>
<td>362</td>
</tr>
<tr>
<td>4</td>
<td>374</td>
</tr>
<tr>
<td>5</td>
<td>373</td>
</tr>
<tr>
<td>6</td>
<td>362</td>
</tr>
<tr>
<td>7</td>
<td>371</td>
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<tr>
<td>8</td>
<td>339</td>
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<td>9</td>
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<td>10</td>
<td>377</td>
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<tr>
<td>11</td>
<td>331</td>
</tr>
<tr>
<td>12</td>
<td>370</td>
</tr>
<tr>
<td>13</td>
<td>371</td>
</tr>
</tbody>
</table>

Table 2: Number of maxima in 500\( \mu \) intervals.

3.2 Mirror displacement

As mirror M1 is moved, the interference goes through maxima and minima. A typical curve for a 5\( \mu m \) displacement is shown in figure 6.

In a larger displacement scale of 600 \( \mu \), the maxima and minima are plotted in figure 7. The maxima and minima so compact in this figure that a solid image is obtained. There are two contributions, small ripples superimposed on a slowly varying visibility. These ripples are in average separated by 27.7 \( \mu \).

On the other hand, the slow variation in the fringe contrast is better appreciated in figure 8, where 4800 \( \mu \) distance is shown. There maxima are not equal but they alternate one high one low. The periodicity between maxima with intensity above 500 A.U. is 1400 \( \mu \). The periodicity between minima is 700 \( \mu \).

Finally, the yellow and red slabs shown in figure 4 were inserted so as to increase the path by 4.0 mm steps. The three curves thus obtained are shown in figure 9. The overall behaviour is similar but the intensity decreases as the path length is increased. This decrease in intensity is not expected and it may be due to slight misalignment introduced by the slabs.
4. DISCUSSION

4.1 Coherence in terms of visibility.

Recall that the intensity of two superimposed fields with intensities $I_1, I_2$ is given by

$$I = I_1 + I_2 + 2\sqrt{I_1I_2} |\gamma| \cos \theta$$

where $|\gamma|$ is the partial degree of coherence and $\theta$ is the angle between the beams. The maxima and minima are then given by $\cos \theta = \pm 1$

$$I_{\text{max}} = I_1 + I_2 + 2\sqrt{I_1I_2} |\gamma|$$

$$I_{\text{min}} = I_1 + I_2 - 2\sqrt{I_1I_2} |\gamma|$$

The visibility in terms of the intensity extrema is defined by
\[ V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \]  

In terms of the beam intensities, the visibility is equal to

\[ V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = \frac{4\sqrt{I_1I_2}}{2(I_1 + I_2)} |\gamma| \]  

(10)

So that the degree of coherence is

\[ |\gamma| = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \left( 1 + \frac{I_2}{I_1} \right) \frac{1}{2\sqrt{\frac{I_2}{I_1}}} \]  

(11)

Figure 7: ripples 600 µm mirror displacement

Figure 8: coherence max-min 5000 µm mirror displacement
The factor involving the intensity ratios, from the data obtained is

\[
\frac{1 + \frac{I_2}{I_1}}{2\sqrt{\frac{I_2}{I_1}}} = \frac{1 + \frac{157.95}{83.25}}{2\sqrt{\frac{157.95}{83.25}}} = 1.0517
\]

so that even though the intensities are far from being 50/50, the impact on the partial degree of coherence is only about 5%.

The coherence of the laser is shown in figure 10. It increases and decreases in an oscillatory fashion as the mirror distance is increased. The kinks are due to the poor overlap obtained when the slabs are introduced and new micrometer runs are performed.

4.2 Autocorrelation theorem

The two-beam fringe visibility due to a light beam of broad spectral distribution is given by the autocorrelation or Wiener-Khintchine theorem – the fringe visibility \( \gamma (\tau) \) and the spectral density function \( s(\nu) \) form a Fourier transform pair

\[
\gamma (\tau) = \int s(\nu) \exp (2\pi i\nu\tau) \, d\nu
\]

\[
s(\nu) = \int \gamma (\tau) \exp (-2\pi i\nu\tau) \, d\tau
\]
If the spectrum consists of a set of discrete lines
\[ s(\nu) = \sum_{n=1}^{N} \delta(\nu - \nu_n) \]

Then \( \gamma(\tau) \) will be given by the sum of cosine functions
\[ \gamma(\tau) = \sum_{n=1}^{N} \cos(2\pi\nu_n\tau) \quad (12) \]

Cosine fringe INTENSITY due to each frequency is summed in Eq.(12). The product \( \nu_n\tau = n\nu \), the order of interference varies for the same differential delay \( \tau \) (mirror displacement). Thus a superposition of translating cosine fringes is obtained and hence the resultant fringe visibility oscillates with \( \tau \).

Let us start from the fundamental principle. If you have \( N \)-frequencies through a Michelson two beam interferometer, then the intensity is given by
\[
I(\tau) = \left| \sum_{n=1}^{N} \exp(i2\pi\nu_n t) + \sum_{m=1}^{N} \exp(i2\pi\nu_m (t + \tau)) \right|^2 \quad (13)
\]
\[
\rightarrow \sum_{n=1}^{N} \left| \exp(i2\pi\nu_n t) + \exp(i2\pi\nu_m (t + \tau)) \right|^2 \quad (14)
\]

Note that the cross-terms between different frequencies, which correspond to heterodyne signals in a fast detector, have been dropped in the second step. In a slow detector, the time average of this heterodyne signal to zero, which is, measurement wise, equivalent to “non-interference between different frequencies”, as was assumed by Michelson for his Fourier transform spectrometry. If the RHS of Eq.(13) is expanded, you will discover Eq.3 is the oscillatory component of this equation without the multiplying constant. If the mode spectrum is multiplied by a Gaussian or Lorentzian spectral envelope function \( b(\nu) \)

\[ s(\nu) = b(\nu) \sum_{n=1}^{N} \delta(\nu - \nu_n) \]

Then, the fringe visibility will be convolved by the FT of the spectral envelope function \( b(\nu) \)

\[ \gamma(\tau) = b(\nu) \otimes \sum_{n=1}^{N} \cos(2\pi\nu_n\tau) \]

It is possible, of course, to start from the fundamental principle, that each spectral amplitude (not intensity) is multiplied by a factor \( a_n \)

\[ I(\tau) = \sum_{n=1}^{N} |a_n \exp(i2\pi\nu_n t) + a_n \exp(i2\pi\nu_m (t + \tau))|^2 \]

The visibility can then be computed from
\[ \gamma(\tau) = \sum_{n=1}^{N} a_n \cos(2\pi\nu_n\tau). \quad (15) \]

Therefore, the discrete modes seen in the Nd:YAG spectrum shown in figure 2 has a time spectrum that is a series of cosine functions. In figure 11, the transform of the sum of two, three and four modes is shown. Clearly, it is a three mode structure is most similar to the experimental results obtained in figure 8.
Figure 11: temporal behaviour of two, three and four modes with equal amplitude

Figure 12: temporal behaviour for three modes, where the second and third modes have 0.8 and 0.6 the first mode amplitude.

However, the minima are not zero as obtained in the previous graph. If different mode amplitudes are considered as proposed in equation (15), then the minima have finite width as shown in figure 12.

The observed temporal behaviour then resemble very closely the experimental data obtained from the Michelson interferometer.

REFERENCES