Charge motion under ultrafast harmonic wave switching

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Abstract. The motion of a free charge subjected to a harmonic electromagnetic wave with transients is analyzed. The two central features of a charge-field interaction with a time dependent wave envelope are the drift or remnant velocity and the charge displacement. The former phenomenon has been observed in above threshold ionization and plasma residual-current density generation by few-cycle laser pulses. The particle shift is a feature that may prove useful for coherent manipulation. Analytical expressions in order to obtain the extrema for the velocity and displacement for a finite gating time are presented.

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INTRODUCTION

Transient effects in electromagnetic waves become relevant in ultrafast gating when the rise / fall time is comparable or smaller than the wave period. Very fast switching devices exist or are being developed in several regions of the electromagnetic spectrum. Transients are also of great importance in ultrafast few cycle optical pulses when the pulse envelope becomes comparable to the wave carrier period. It is now standard for example, to produce pulses centered around 760 nm (2.7 fs carrier period) with 6 fs duration using Ti:sapphire laser systems with subsequent hollow-core fibre broadening and chirped compression [1]. Less than two oscillation cycles down to single cycle pulses have been produced with several schemes, notably through the coherent superposition of two colour ultrabroadband laser fields [2]. Production and detection of sub cycle pulses is also being accomplished in attosecond science [3, 4].

Very fast optical gating is being used to study intense light-matter interaction [5]. Half cycle pulses have been used to probe and ionize wave packets in atomic systems [6, 7, 8]. Electron acceleration in vacuum with harmonic electromagnetic fields under different regimes has been predicted and demonstrated [9, 10, 11, 12]. Precise understanding and control of the carrier envelope phase in few cycle pulses is important for diverse purposes such as attosecond metrology and spectroscopy [13], coherent control of molecular and charge dynamics as well as ultrafast nonlinear optics.

One of the most fundamental radiation and matter interactions is that of a charge in a harmonic electromagnetic field [14]. This problem is modeled, if the charge is free, by the differential equation for the charge motion subjected to an external force produced
FIGURE 1. Free charge - electromagnetic field interaction. A flagship phenomenon is shown for each regime. The frontier between the weak non-relativistic regime and the strong field relativistic regime has been somewhat arbitrarily set around the petawatt power densities. The frontier between long, short and ultrafast gating is dependent on the region of the electromagnetic spectrum. The relevant quantity is the carrier envelope ratio (CER) introduced in equation (5).

by the electromagnetic field. It is on the other hand, described with a quantum treatment if the charge is bound. The harmonic field - charge interaction has been tackled under different conditions: Classical or relativistic particle description depending on the velocities attained by the charge. In the field counterpart, only the electric field contribution of the Lorentz force is considered in the former case. Whereas, the magnetic contribution needs to be taken into account when the particle velocity becomes relativistic. On the other hand, the field amplitude can be i) constant as in CW laser sources; ii) vary adiabatically or slowly over many cycles as in nano or pico second pulses in the visible region; or iii-a) vary from zero to its maximum value in a few cycles as in ultrafast femtosecond or attosecond pulses; iii-b) gated or switched in times much shorter than the wave period, regardless of the time elapsed between gatings. These criteria are depicted in figure 1.

The effect of the field transient on the motion of a charge is the central purpose of this communication. Different temporal pulse envelope shapes such as Gaussian, hyperbolic
secant or cardinal cosine are common in the literature [15, Hirlimann Ch.2] giving rise to apparently different features [16, p.17]. The envelope maximum slope and the pulse duration are tied together for each type of shape. In order to separate these effects as well as allowing for optical gating, the present approach considers independent switch on, pulse duration and switch off times.

Two universal phenomena due to transient harmonic field - charge interaction are obtained:

1. a drift velocity when the field is turned-on or remnant velocity when it is turned-off,
2. a shift or displacement of the charge position.

Even in a continuous wave or a slowly varying turn on time it is possible to obtain analogous results if the charges are suddenly placed in the presence of the field. That is, rather than switching on the field very rapidly it is the charge that is placed very rapidly under the field influence. Such is the case in above threshold ionization (ATI) experiments where an electron is ionized and suddenly placed under the influence of the field. A quiver movement and a constant drift are then characteristic to the charge motion [17, 18].

The present description is performed within a low field approximation where the particle may be classically modeled and only the electric field contribution to the Lorentz force is considered. This scenario corresponds to the upper half of figure 1. The treatment is restricted to one dimension, that is, a plane wave formalism in order to exhibit the key features and obtain closed analytical solutions. Three dimensional vector solutions have been obtained for Gaussian pulses based on a complex source approach [19]. The simple model presented here allow us to evaluate the maximum velocity drift for finite switch on/off times.

**FIELD WITH EXPONENTIAL GROWTH**

Consider one of the simplest cases of a switch on harmonic field. Allow for a plane wave field that grows from zero to an asymptotic amplitude $A_0$ in an exponential fashion

$$E(t) = A_0 \left(1 - e^{-\beta_{on} t}\right) \cos(\omega t + \phi_{CE}), \quad t \geq 0$$

where the rate $\beta_{on} \geq 0$ is a measure of how fast the field is build up and $\phi_{CE}$ establishes the initial phase between the carrier wave at frequency $\omega$ and the wave envelope. Let this plane wave be incident on a free charge $q$ with mass $m$. Only the electric field term of the Lorentz force is evaluated. The magnetic term being dismissed by assuming non relativistic velocities. The trajectory of the charge from classical mechanics is then described by the differential equation

$$m \ddot{x} = m \dot{v} = qA_0 \left(1 - e^{-\beta_{on} t}\right) \cos(\omega t + \phi_{CE}), \quad (1)$$

valid for $t \geq 0$. 

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The first integral to this equation is

\[
v(t) = \frac{A_0 q}{m} \left[ \left( \frac{1}{\omega} - \frac{\omega e^{-\beta_{on} t}}{(\beta_{on}^2 + \omega^2)} \right) \sin(\omega t + \phi_{CE}) + \frac{\beta_{on} e^{-\beta_{on} t}}{(\beta_{on}^2 + \omega^2)} \cos(\omega t + \phi_{CE}) \right] + v_d
\]

If the velocity at an initial time is \(v(t=0) = v_0\), the constant velocity \(v_d\) is

\[
v_d = v_0 - \frac{A_0 q}{m \omega} \left( \frac{1}{1 + \frac{\omega^2}{\beta_{on}^2}} \right) \left[ \sin(\phi_{CE}) + \frac{\omega}{\beta_{on}} \cos(\phi_{CE}) \right], \tag{2}
\]

thus \(v_d\) is the sum of the initial velocity before the field is turned on plus the constant velocity due to the transient. The particle velocity is then

\[
v(t) = \frac{A_0 q}{m \omega} \left( \frac{1}{1 + \frac{\omega^2}{\beta_{on}^2}} \right) \left[ \left( 1 + \zeta_{on}^2 \right) \left( 1 - e^{-\beta_{on} t} \right) \right] \sin(\omega t + \phi_{CE}) + \frac{\beta_{on} e^{-\beta_{on} t}}{(\beta_{on}^2 + \omega^2)} \cos(\omega t + \phi_{CE}) \right] - \frac{A_0 q}{m \omega} \left( \frac{1}{1 + \frac{\omega^2}{\beta_{on}^2}} \right) \sin(\phi_{CE}) + \zeta_{on} \cos(\phi_{CE}) \right] + v_0. \tag{3}
\]

where the carrier envelope ratio (CER)

\[
\zeta_{on} \equiv \frac{\omega}{\beta_{on}} \tag{5}
\]

is a measure of the envelope risetime in relation to the carrier wave period. The charge exhibits two superimposed contributions: i) An oscillatory velocity and, ii) a constant velocity drift that depends on the \(\frac{\omega}{\beta_{on}}\) quotient and the initial carrier envelope phase \(\phi_{CE}\). A typical plot is shown in figure 2.

**velocity: abrupt, steady state and adiabatic limits**

In order to assess these two terms consider three limits of this expression, the abrupt limit, the adiabatic approximation and the steady state solution. The abrupt limit is obtained for \(\beta_{on} \gg \omega\),

\[
\lim_{\frac{\omega}{\beta_{on}} \to 0} v(t) = \frac{A_0 q}{m \omega} \left[ \sin(\omega t + \phi_{CE}) - \sin(\phi_{CE}) \right] + v_0
\]

Therefore, even if the charge is initially at rest, it acquires a drift velocity whose magnitude is dependent on the specific phase \(\phi_{CE}\) of the harmonic wave when the field is
FIGURE 2. The electric field with $A_0 = 1$ (filled curve) and its envelope (dashed curve) are shown as a function of time in the vicinity of the turn on transient. The charge velocity with $\frac{A_0 q}{m \omega} = 1$ (purple curve) and the charge position (green curve) are also shown. This plot exhibits a negative drift velocity superimposed on the oscillatory behaviour, $\zeta_{on} = \frac{\omega}{\beta_{on}} = 2, \phi_{CE} \approx 0.46$.

abruptly turned on. This result, as expected, is the solution to the differential equation (1) with a constant amplitude harmonic driving source. This expression has been extensively used in the above threshold ionization (ATI) quasi-static model [17, 20, 21].

For sufficiently long times $t \to \infty$, then $e^{-\beta t} \to 0$ and the steady state solution is obtained

$$v(t) = \lim_{t \to \infty} v(t) = A_0 q \left[ \sin(\omega t + \phi_{CE}) - \frac{1}{1 + \zeta_{on}^2} \sin(\phi_{CE}) \cos(\phi_{CE}) \right] + v_0.$$  

The velocity of the charge is thus dependent on the way the field was built up even if the charge motion is being evaluated at much longer times when the field amplitude is already constant. The charge velocity depends on the present value of the field and the value of the field at a much earlier time when the field began to rise.

The slowly varying envelope limit is obtained for $\beta_{on} \ll \omega$, The velocity is then

$$v(t) = \lim_{\zeta_{on} \to \infty} v(t) = A_0 q \left[ (1 - e^{-\beta_{on} t}) \sin(\omega t + \phi_{CE}) \right] + v_0.$$  

The velocity of the charge in the adiabatic limit follows the driving field with a lag of $\frac{\pi}{2}$. The carrier envelope phase $\phi_{CE}$ only enters the solution as an initial phase where the time origin is chosen. The steady state solution in this slowly varying envelope limit is

$$v(t) = \lim_{t, \zeta_{on} \to \infty} v(t) = A_0 q \left[ \sin(\omega t + \phi_{CE}) \right] + v_0.$$  

velocity: zero drift

We have the picture that a harmonic field will wiggle a free electron about its original position but will not produce a net displacement. However, an electron originally at rest.
\( \nu_0 = 0 \) that is subjected to a harmonic field will execute an oscillatory motion about its original position only if the drift velocity \( \nu_d \) is zero. From (2), this condition is fulfilled if:

1. the field is turned on adiabatically, that is the factor \( (1 + \zeta_{on}^2)^{-1} \) in the drift term tends to zero, or
2. the carrier envelope phase is \( \phi_{CE} = -\arctan(\zeta_{on}) \)

In the abrupt limit \( \zeta_{on} = \frac{\nu_0}{\nu_{on}} \to 0 \), this last condition implies that the CEP must be zero. Therefore, in order to have a zero drift velocity for an electron originally at rest if the harmonic field gating is abrupt, it has to be suddenly turned on to its maximum value in a time much shorter than the period of oscillation.

In some regions of the electromagnetic spectrum such fast gatings are difficult to accomplish. For example, attosecond gatings for terahertz radiation. However, above threshold ionization (ATI) achieves the same effect in the following way: The field may not have a very fast risetime or could even be continuous. An atom containing a bound electron is exposed to the external field. However, the charge is “concealed” from the field within the atom. Due to the field, the electron is ionized and when just ionized it is at rest and suddenly exposed to the field.

On the other hand, if the abrupt pulse begins when the envelope passes through zero, the carrier envelope phase is \( \phi_{CE} = \frac{\pi}{2} \). It is not possible to have zero drift velocity if the charge is initially at rest \( \nu_0 = 0 \), since then \( \nu_d = \frac{A_0q}{m\omega} \). In order to have zero drift, the electron has to have an initial velocity before the pulse begins, carrier envelope ratio and carrier envelope phase is

\[
\nu_d = 0 \implies \nu_0 = \frac{A_0q}{m\omega} \left[ \sin(\phi_{CE}) + \frac{\nu_{on}}{\zeta_{on}} \cos(\phi_{CE}) \right].
\]

**velocity: maximum drift**

The extrema of the normalized drift velocity function

\[
\nu_{dn} = (\nu_d - \nu_0) \frac{m\omega}{A_0q} = -\frac{1}{(1 + \zeta_{on}^2)} \left[ \sin(\phi_{CE}) + \frac{\nu_{on}}{\zeta_{on}} \cos(\phi_{CE}) \right],
\]

are obtained by equating to zero its derivative with respect to the carrier to envelope phase (CEP). For a carrier envelope ratio \( \zeta_{on} \), the largest velocity drift is obtained for a CEP \( \phi_{CE} \) given by

\[
\phi_{CE} = \arccos\left( \frac{\zeta_{on}}{\sqrt{1 + \zeta_{on}^2}} \right) + N\pi,
\]

\(62\)
FIGURE 3. The left hand plot shows the relationship between the carrier envelope ratio $\zeta_{\text{on}} = \frac{\omega}{\beta}$ and carrier envelope phase CEP when the drift velocity is maximized. The log-linear plot on the right depicts the normalized maximum drift velocity $v_{dn}^{(\text{max})} = \frac{1}{\sqrt{1 + \zeta_{\text{on}}^2}}$ as a function of the CER $\zeta_{\text{on}}$. The drift velocity tends to zero in the slowly varying amplitude (adiabatic) limit.

where $N \in \mathbb{N}$. Substitution of this phase in the drift velocity expression gives the maximum possible drift as a function of carrier envelope ratio

$$v_d^{(\text{max})} = v_0 + \frac{A_0 q}{m \omega} \frac{1}{\sqrt{1 + \zeta_{\text{on}}^2}}. \quad (9)$$

The plots of these expressions are shown in figure 3. For example, if the risetime to 63% $(1 - \frac{1}{e})$ takes place in a third of a period, the CER is $\zeta_{\text{on}} \approx 2$. The maximum normalized drift velocity is then $v_{dn} = -\frac{1}{\sqrt{5}} \approx -0.45$ if the CEP is optimized to $\phi_{\text{CE}} = \arccos \left( \frac{2}{\sqrt{5}} \right) \approx 0.46$. The transient field and the corresponding velocity for these parameters is depicted in figure 2.

In the slowly varying envelope limit $v_d^{(\text{max})}$ decreases as $\zeta^{-1}$. This approximation is already quite good for a 63% risetime at two carrier periods ($\zeta_{\text{on}} = 4\pi$) where the exact value is $v_{dn}^{(\text{max})} = 0.07933$ whereas $\zeta^{-1} = 0.07958$ is the approximate value. Turning on the field at a rate much faster that the oscillation period is close to the abrupt switch on limit. The gate bandwidth defines how fast the field may be commuted.

**position switch on solution**

The second integral of the differential equation gives the position

$$x(t) = \frac{A_0 q}{m \omega^2} \frac{1}{1 + \zeta_{\text{on}}^2} \left[ 2 r_{\text{on}}^3 e^{-\beta_{\text{on}} t} \sin(\omega t + \phi_{\text{CE}}) - \left( 1 + \zeta_{\text{on}}^2 \right)^2 + \zeta_{\text{on}}^2 (1 - r_{\text{on}}^2) e^{-\beta_{\text{on}} t} \right] + v_d t + x_d$$
where \( v_d \) is given by 2. At \( t = 0 \), the position is

\[
x(0) = \frac{A_0 q}{m \omega^2} \frac{1}{(1 + \xi_{on}^2)^2} \left[ 2 \xi_{on}^3 \sin (\phi_{CE}) - (1 + 3 \xi_{on}^2) \cos (\phi_{CE}) \right] + x_d;
\]

If the initial position is \( x(0) = x_0 \), the integration constant gives the displacement constant

\[
x_d = x_0 - \frac{A_0 q}{m \omega^2} \frac{1}{(1 + \xi_{on}^2)^2} \left[ 2 \xi_{on}^3 \sin (\phi_{CE}) - (1 + 3 \xi_{on}^2) \cos (\phi_{CE}) \right].
\]

The charge trajectory is comprised by three terms, an oscillatory motion (in turn, composed by in and out of phase terms), a linear time displacement due to the drift velocity and a constant shift.

To elucidate the last term, consider the ultrafast gating limit

\[
\lim_{\xi_{on} \to 0} x(t) = \frac{A_0 q}{m \omega^2} \left[ -\cos(\omega t + \phi_{CE}) - \omega \sin (\phi_{CE}) t + \sin (\phi_{CE}) \right] + v_0 t + x_0.
\]

Allow for a charge originally at rest at the origin \( v_0 = x_0 = 0 \) and zero carrier envelope phase \( \phi_{CE} = 0 \) so that the drift velocity term (linear with time) vanishes. In this abrupt case the charge oscillates about the \( -\frac{A_0 q}{m \omega^2} \) position rather than the original \( x = x_0 = 0 \) position. This net displacement is due to the initial transient of the field. The maximum displacement is obtained by equating to zero the position’s derivative with respect to the carrier to envelope phase (CEP). For a carrier envelope ratio \( \xi_{on} \),

\[
\frac{\partial}{\partial \phi_{CE}} \left[ (x_d - x_0) - \frac{m \omega^2}{A_0 q} \right] = -\frac{1}{(1 + \xi_{on}^2)^2} \left[ 2 \xi_{on}^3 \cos (\phi_{CE}) + (1 + 3 \xi_{on}^2) \sin (\phi_{CE}) \right] = 0.
\]

then the largest velocity drift is obtained for a CEP \( \phi_{CE} \) given by

\[
\phi_{CE} = \arctan \left( -\frac{2 \xi_{on}^3}{1 + 3 \xi_{on}^2} \right).
\]

Substitution of this phase in the drift velocity expression gives the maximum possible drift as a function of carrier envelope ratio

\[
x_d^{(max)} = x_0 - \frac{A_0 q}{m \omega^2} \frac{1}{(1 + \xi_{on}^2)^2} \left[ 2 \xi_{on}^3 \sin (\phi_{CE}) - (1 + 3 \xi_{on}^2) \cos (\phi_{CE}) \right] + x_0.
\]

The steady state solution is

\[
\lim_{t \to \infty} x(t) = -\frac{A_0 q}{m \omega^2} \cos(\omega t + \phi_{CE}) - \frac{A_0 q}{m \omega} \frac{1}{(1 + \xi_{on}^2)^2} (\sin (\phi_{CE}) - \xi_{on} \cos (\phi_{CE})) t + v_0 t
\]

\[
-\frac{A_0 q}{m \omega^2} \frac{1}{(1 + \xi_{on}^2)^2} \left[ 2 \xi_{on}^3 \sin (\phi_{CE}) - (1 + 3 \xi_{on}^2) \cos (\phi_{CE}) \right] + x_0.
\]
The adiabatic approximation gives
\[
\lim_{\zeta \to \infty} x(t) = -\frac{A_0 q}{m \omega^2} \left[ \left( 1 - e^{-\beta_{off} t} \right) \cos (\omega t + \phi_{CE}) \right] + v_0 t + x_0.
\]

The steady state of this adiabatic limit for a charge originally at rest at the origin \(v_0 = x_0 = 0\) is
\[
\lim_{\zeta, t \to \infty} x(t) = -\frac{A_0 q}{m \omega^2} \cos (\omega t + \phi_{CE})
\]

This is the solution that we are accustomed to see with little regard as to how it was obtained.

**proposition**

The solution \(v(t)\) to the differential equation \(m \ddot{v} = f(t) \cos (\omega t + \phi_{CE})\) when \(\frac{f'(t)}{f(t)} \ll \omega\) is proportional to \(\int \cos (\omega t + \phi_{CE})\).

**SWITCH OFF TIMES**

The differential equation for a field that is turned off in an exponential fashion at a time \(t_{off}\) is
\[
m \ddot{x} = m \dot{v} = q A_0 e^{-\beta_{off} (t - t_{off})} \cos (\omega t + \phi_{CE}) \quad t \geq t_{off}.
\]
The ODE solution is
\[
v(t) = \frac{A_0 q e^{-\beta_{off} (t - t_{off})}}{m \omega} \left( \frac{\zeta_{off}^2}{1 + \zeta_{off}^2} \right) \left( \zeta_{off}^2 \sin (\omega t + \phi_{CE}) - \zeta_{off} \cos (\omega t + \phi_{CE}) \right) + v_d,
\]
where the exponential decay carrier envelope ratio (ED-CER) is
\[
\zeta_{off} \equiv \frac{\omega}{\beta_{off}}.
\]

To evaluate the constant \(v_d\), it is necessary to establish the charge velocity at time \(t_{off}\). This velocity in turn depends on the way the field was switched on. Let us examine a case where the source of the drift velocity and displacement arise solely from the switch off time. Form the previous study this is so if we consider an adiabatic switch on time and consider a long time after having turned on the field. The velocity from (6) is then
\[
v(t_{off}) = \frac{A_0 q}{m \omega} \left[ \sin (\omega t_{off} + \phi_{CE}) \right] + v_0
\]
Equating this result with the above ODE solution at \(t = t_{off}\) guarantees that a smooth solution is obtained for all times. The constant \(v_d\) is thus
\[
v_d = \frac{A_0 q}{m \omega} \left( \frac{1}{1 + \zeta_{off}^2} \right) \left[ \sin (\omega t_{off} + \phi_{CE}) + \zeta_{off} \cos (\omega t_{off} + \phi_{CE}) \right] + v_0
\]
The velocity for times \( t \geq t_{\text{off}} \) is then

\[
v(t \geq t_{\text{off}}) = \frac{A_0 q}{m\omega} \left(\frac{1}{1 + \zeta_{\text{off}}^2}\right) e^{-\beta_{\text{off}}(t-t_{\text{off}})} \left(\zeta_{\text{off}}^2 \sin(\omega t + \phi_{\text{CE}}) - \zeta_{\text{off}} \cos(\omega t + \phi_{\text{CE}})\right) + \sin(\omega t_{\text{off}} + \phi_{\text{CE}}) + \zeta_{\text{off}} \cos(\omega t_{\text{off}} + \phi_{\text{CE}}), \tag{12}\]

where the charge velocity before the pulse arrival has been set to zero.

**Switch off: abrupt, adiabatic and steady state limits**

In order to assess these two terms consider three limits of this expression, the abrupt limit, the adiabatic approximation and the steady state solution. The abrupt limit is obtained for \( \zeta_{\text{off}} \ll 1 \) \((\beta_{\text{off}} \gg \omega)\),

\[
\lim_{\omega \beta \to 0} v(t) = \frac{A_0 q}{m\omega} \left[ \sin(\omega t_{\text{off}} + \phi_{\text{CE}}) \right]
\]

Therefore, depending on the switch off time, the charge exhibits a remnant constant velocity whose magnitude is dependent on the specific phase \( \phi_{\text{CE}} \) of the harmonic wave when the field is abruptly turned off.

For sufficiently long times \( t \to \infty \), then \( e^{-\beta_{\text{off}}(t-t_{\text{off}})} \to 0 \) and the solution long after the pulse is switched off is

\[
\lim_{t \to \infty} v(t) = \frac{A_0 q}{m\omega} \left(\frac{1}{1 + \zeta_{\text{off}}^2}\right) \left[ \sin(\omega t_{\text{off}} + \phi_{\text{CE}}) + \zeta_{\text{off}} \cos(\omega t_{\text{off}} + \phi_{\text{CE}})\right].
\]

The velocity of the charge is thus dependent on the way the field was built up even if the charge motion is being evaluated at much longer times when the field amplitude is already constant. The charge velocity depends on the present value of the field and the value of the field at a much earlier time when the field began to rise.

The slowly varying envelope limit is obtained for \( \zeta_{\text{off}} \gg 1 \) \((\beta_{\text{off}} \ll \omega)\). The velocity is then

\[
\lim_{\zeta_{\text{off}} \to \infty} v(t) = v(t) = \frac{A_0 q}{m\omega} \left[ e^{-\beta_{\text{off}}(t-t_{\text{off}})} \sin(\omega t + \phi_{\text{CE}}) \right].
\]

The velocity of the charge in the adiabatic limit follows the driving field with a lag of \( \pi/2 \). The carrier envelope phase \( \phi_{\text{CE}} \) only enters the solution as an initial phase where the time origin is chosen. The steady state solution in this slowly varying envelope limit is zero

\[
\lim_{t, \zeta_{\text{off}} \to \infty} v(t) = 0. \tag{13}
\]

The zero drift velocity \( (v_d = 0) \), is obtained from (11) either if the switch off time is adiabatic \( \zeta_{\text{off}} \to \infty \) or the CEP and the CER satisfy the relationship

\[
\phi_{\text{CE}} = -\omega t_{\text{off}} - \arctan(\zeta_{\text{off}}).
\]
The maxima of the normalized drift velocity function

\[ v_{dn} = (v_d - v_0) \frac{m\omega}{A_0q} = \frac{1}{\left(1 + \zeta_{off}^2\right)} \left[ \sin(\omega t_{off} + \phi_{CE}) + \zeta_{off} \cos(\omega t_{off} + \phi_{CE}) \right], \]

are obtained by equating to zero its derivative with respect to the carrier to envelope phase (CEP).

\[ \frac{\partial v_{dn}}{\partial \phi_{CE}} = \frac{1}{\left(1 + \zeta_{off}^2\right)} \left[ \cos(\omega t_{off} + \phi_{CE}) - \zeta_{off} \sin(\omega t_{off} + \phi_{CE}) \right], \]

equate to zero

\[ \phi_{CE} = \arctan \left( \frac{1}{\zeta_{off}} \right) - \omega t_{off}, \]

For a carrier envelope ratio \( \zeta_{off} \), the largest velocity drift is obtained for a CEP \( \phi_{CE} \) given by

\[ \phi_{CE} = \arccos \left( \frac{\zeta_{off}}{\sqrt{1 + \zeta_{off}^2}} \right) - \omega t_{off} + N\pi, \quad (14) \]

where \( N \in \mathbb{N} \). Substitution of this phase in the drift velocity expression gives the maximum possible drift as a function of carrier envelope ratio

\[ v_{dn} = \frac{1}{\left(1 + \zeta_{off}^2\right)} \left[ \frac{1}{\sqrt{1 + \zeta_{off}^2}} + \zeta_{off} \frac{\zeta_{off}}{\sqrt{1 + \zeta_{off}^2}} \right] \frac{1}{\sqrt{1 + \zeta_{off}^2}} = \frac{1}{\sqrt{1 + \zeta_{off}^2}}, \]

then

\[ v_{d}^{(max)} = v_0 + \frac{A_0q}{m\omega} \frac{1}{\sqrt{1 + \zeta_{off}^2}}. \quad (15) \]

**CONCLUSIONS**

The interaction between a charge and a carrier wave with exponential switch on and switch off envelopes has been described in the non relativistic regime. Two phenomena occur due to a transient harmonic field - charge interaction: i) a drift velocity when the field is turned-on or remnant velocity when it is turned-off and ii) a shift or displacement of the charge position.

The transient behaviour is usually treated in two limits, the adiabatic and the abrupt limits. However, as we approach the abrupt limit what is actually achieved experimentally is a fast rise or fall time that is comparable with the carrier period. The carrier envelope ratio (CER), defined as the ratio of carrier frequency over exponential growth rate (5), has been introduced as a measure of how fast the field transient takes place. The maximum velocity drift is obtained for a carrier envelope phase (CEP) given by (14). In
the abrupt limit, the CER is zero and the maximum drift is obtained when the CEP is $\pi/2$. As the pulse gating is slower, the maximum remnant velocity decreases as may be seen from (15). The maximum velocity is obtained for CEP values that depart from $\pi/2$ and move towards zero.

The adiabatic or slowly varying field envelope variation has been shown to be equivalent to an abrupt field scenario where the field starts with zero amplitude and the charge has a finite initial velocity or the field starts with maximum amplitude and the charge is at rest (intermediate cases are possible). It is often argued that transients can be dismissed long after the transient took place once the steady state has been obtained. However, transients can be ignored only if the transients are adiabatic. When the transient is comparable or shorter than the carrier wave it produces a remnant velocity that is present even long after the transient took place. The charge trajectory becomes non-local in time since it can depend on much earlier times when a transient took place. In other words, the charge retains a memory of the way the field was switched on or off.

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